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COMPARISON OF BUTT TENSILE STRENGTH DATA WITH INTERFACE CORNER STRESS INTENSITY FACTOR PREDICTION

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Abstract—The butt tensile strength of a joint that bonds two stainless steel rods together with an unfilled epoxy adhesive (Epon 828/T-403) has been determined for a wide range of bond thicknesses. The measured joint strength shows a pronounced bond thickness dependence; joint strength increases by a factor of 2 as bond thickness is reduced from 2.0 to 0.25 mm. A failure criterion, based upon a critical interface corner stress intensity factor, accurately predicts the observed bond thickness effect. This fracture criterion suggests that the strength of an adhesively bonded butt tensile joint of one bond thickness can be estimated from strength data for a joint with a different bond thickness by the simple relation

$$\sigma_{t2}^{\mathrm{ult}} = \sigma_{t1}^{\mathrm{ult}}(h_1/h_2)^{1/3},$$

where $2h_i$ is bond thickness, $\sigma_{ti}^{\mathrm{ult}}$ is the nominal butt tensile strength, and subscript $i = 1, 2$ identifies the two joints with differing bond thickness. This relation applies to thin bonds when the adhesive's Poisson's ratio is between 0.3 and 0.4, the adherends are relatively stiff, and small scale yielding conditions hold at the interface corner.

INTRODUCTION

Adhesively bonded joints are widely used in the aerospace and automotive industries, and there appears to be a growing interest in using bonded joints in structurally demanding applications (Clark, 1990; Drain and Chandrasekharan, 1990). Light-weight materials, such as polymer matrix composites and aluminum, are being used more and more as a result of ever increasing performance and energy efficiency requirements. Traditional joining techniques, such as welding, are frequently inapplicable, and it can be argued that adhesive bonding is often the best method for joining such materials. Accordingly, methods for predicting the strength, reliability and durability of adhesively bonded joints is of increasing interest to designers. A review of the published literature suggests that there are three widely accepted methods for predicting the strength of bonded joints. One approach uses approximate, shear-lag-based, elastic–plastic stress analyses for bond stress and strain. Joint failure is predicted to occur at a critical adhesive shear strain (Hart-Smith, 1981). In another approach, detailed finite element analyses of the joint are performed with the adhesive modeled as an unflawed (uncracked), elastic–plastic material. It has been suggested that a maximum principal stress criterion works best for brittle adhesives, while a maximum principal strain condition should be used for toughened adhesives (Adams and Wake, 1984). Linear elastic fracture mechanics concepts have also been applied to bonded joints. A variety of adhesively bonded fracture specimens have been developed to measure the Mode I, Mode II and mixed Mode I and II adhesive fracture toughness (Liechti, 1990). Fracture mechanics-based approaches for predicting joint failure are still evolving. For example, it has been reported by Anderson and DeVries (1989) that a method that utilizes both a critical energy release rate and an inherent flaw size has successfully correlated the failure of butt tensile joints.

In addition to those methods for predicting joint failure discussed above, there may be circumstances where an interface corner stress intensity factor-based approach might apply. Within the context of elasticity theory, a stress singularity of type Kr^{δ} $(\delta < 0)$ can exist at an interface corner (e.g. the point where an interface between bonded materials intersects a stress-free edge) (Williams, 1952). The magnitude of the stress intensity factor K characterizes the stress state in the region of the interface corner. Several experimental studies have investigated the use of an interface corner stress intensity factor to predict the failure of bonded materials. Gradin (1982) tested three different types of 3-layer, steel/epoxy/steel model laminates subjected to various loading conditions. The epoxy layer joining the steel adherends in these model laminates is a relatively thick 25 mm. The reported agreement between test and analysis is fair. Groth (1988) tested single-lap joints with a spew fillet for a range of overlap lengths. The agreement between test and analysis is good for large overlap lengths, but is rather poor for smaller overlaps. Hattori *et al.* (1989) reported that they successfully predicted cooling induced delamination failures in molded epoxy models containing small Fe-Ni inserts. The data reported in all three studies lend support to a failure analysis based upon an interface corner stress intensity factor.

In recent work, the interface corner stress intensity factor for a thin linear elastic adhesive layer bonded to rigid adherends has been fully determined for a transverse tension loading (Reedy, 1990). K_f was determined by a technique that combines results of an asymptotic stress singularity analysis with those of a detailed finite element analysis. This stress intensity factor, referred to here as the free-edge stress intensity factor K_f, is applicable to both plane strain and axisymmetric geometries. It applies when the adherends are much stiffer than the adhesive, as is the case of steel adherends and epoxy adhesive. The geometry considered models adhesive butt tensile test configurations that bond a thin adhesive layer between two relatively rigid metal cylinders (e.g. ASTM D897-78 and D2095-72, 1990 Annual Book of ASTM Standards, Volume 15.06 Adhesives).

To be useful as a failure criterion, the asymptotic stress state characterized by K_f must dominate a region about the interface corner that is significantly larger than the fracture process zone, intrinsic flaw size, and the plastic yield zone. Results of detailed elastic–plastic finite element analyses of a thin adhesive layer subjected to a butt tensile loading and with properties representative of a high strength epoxy have established that the region dominated by the interface corner stress singularity is reasonably large relative to layer thickness, and that the interface corner plastic yield zone is contained within the asymptotic field at nominal failure loads (Reedy, 1993). These calculated results suggest that it may be possible to characterize butt tensile joint failure in terms of a critical value of K_f, referred to here as the interface corner fracture toughness K_{fc}. In the following, butt tensile test results for joints with adhesive thickness ranging from 0.25 to 2.0 mm are reported, and these data are then compared with a K_{fc}-based prediction.

INTERFACE CORNER STRESS INTENSITY FACTOR K_f

A stress singularity exists at the interface corner between bonded elastic and rigid quarter planes (Williams, 1952). The stress components in the region dominated by the stress singularity are

$$\sigma_r = K_f r^{\lambda-1} f_r(\theta), \tag{1}$$

$$\sigma_\theta = K_f r^{\lambda-1} f_\theta(\theta), \tag{2}$$

$$\sigma_{r\theta} = K_f r^{\lambda-1} f_{r\theta}(\theta), \tag{3}$$

where the interface corner coordinate system is defined in Fig. 1 (note $\theta = 0$ corresponds to the interface, while $\theta = -\pi/2$ corresponds to the stress-free edge). K_f is defined so that the stress component normal to the interface $\sigma_\theta(r, 0)$ equals $K_f r^{\lambda-1}$. The order of the stress singularity, $\lambda - 1$, is fully determined by the asymptotic singularity analysis and depends only on Poisson's ratio ν for bonded elastic and rigid quarter planes. Likewise, the functions $f_r(\theta), f_\theta(\theta)$ and $f_{r\theta}(\theta)$ depend only on ν, and are fully determined by the asymptotic analysis (since these are lengthy expressions, they will not be listed here). The interface corner stress intensity factor K_f is the only quantity not determined by the asymptotic analysis. K_f is determined by the full solution, and it depends on loading, geometry and layer elastic properties. The value of K_f characterizes the magnitude of the stress state in the region of the interface corner.

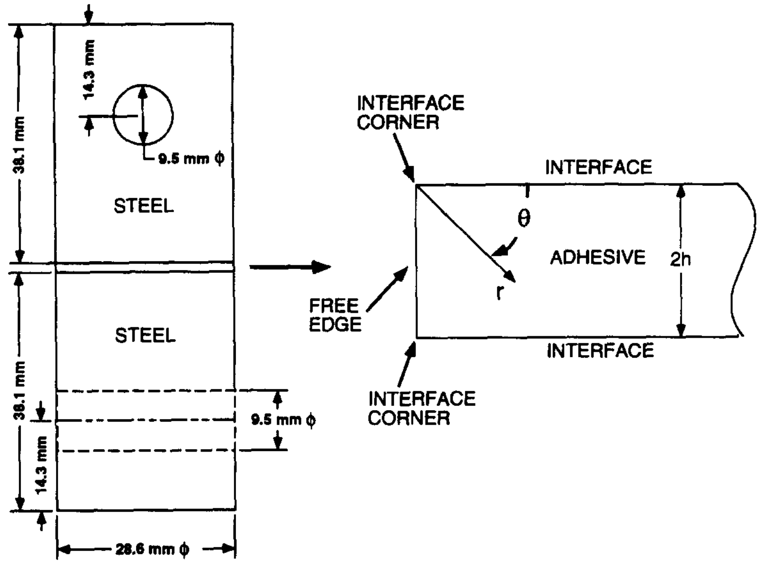

Fig. 1. Schematic of the cylindrical butt tensile joint tested.

The K_f relation for a thin linear elastic layer bonded to rigid adherends has been determined for a transverse tension (butt tensile) loading by Reedy (1990). A layer is considered thin when the stress state in the center of the layer is uniform and unaffected by the stress-free edge. Consequently, the layer behaves as if it is semi-infinite, and the only finite geometric length scale is layer thickness $2h$. This K_f relation is applicable to both plane strain and axisymmetric geometries, and takes the form

$$K_f = \sigma^* h^{1-\lambda} A_p(v), \tag{4}$$

where σ^* is a characteristic stress, $2h$ is layer thickness, and $A_p(v)$ is a function of Poisson's ratio v. For a typical epoxy adhesive with $v = 0.35$, $\lambda - 1 = -0.32$ and $A_p(0.35) = 0.948$. Note, $\lambda - 1$ and $A_p(v)$ values are tabulated for the full range of Poisson's ratio in Reedy (1990, 1993). The characteristic stress σ^* equals the uniform in-plane stress developed within the layer's interior (remote from the stress-free edge). The characteristic stress associated with a nominal applied transverse (butt tensile) stress σ_t^* is

$$\sigma^* = \left(\frac{v}{1-v}\right)\sigma_t^*. \tag{5}$$

A K_{fc} fracture criterion can only apply if small scale yielding conditions hold at the interface corner. An estimate for the length of the finger-like plastic yield zone, r_p, that grows from the interface corner (Reedy, 1993) is

$$r_p = CK_f^{1/(1-\lambda)}, \tag{6}$$

where

$$C = 1.5(\sigma_y/f_e(\theta))^{1/(\lambda-1)}. \tag{7}$$

Since elastic–plastic finite element solutions indicate maximum yielding at $\theta = -\pi/6$, $\theta = -\pi/6$ is used in eqn (7) (note, $f_e(\theta)$ defines the angular dependence of effective stress in the region dominated by the interface corner stress singularity, and for $v = 0.35$, $f_e(-\pi/6) = 0.966$).

BUTT TENSILE STRENGTH TESTS

A schematic of the butt tensile joint specimen is shown in Fig. 1. In essence, the joint specimen bonds two 303 stainless steel rods together with an unfilled epoxy adhesive. The stainless steel adherends are solid cylinders (28.6 mm diameter by 38.1 mm long) that have been precision machined to guarantee that the ends are flat and perpendicular to the cylinder axis. A transverse hole drilled through each adherend (9.5 mm diameter, 14.3 mm from the non-bonded end) is used to pin the joint specimen into the load train for testing. The adhesive is a mix of Shell Epon 828 epoxy resin and Texaco T-403 hardener (100/36 weight ratio), and is cured at room temperature for more than seven days. A room temperature cure is used to minimize residual fabrication stresses. Figure 2 plots the adhesive's measured tensile and compressive stress strain relation. Adhesive properties were measured using strain gaged, cast dog-bone specimens tested in tension, and cylindrical specimens (height/diameter = 2) tested in axial compression (applied strain rate $\dot{\varepsilon} \sim 0.0003\ s^{-1}$). The adhesive's measured Young's modulus is 3.5 GPa, and its Poisson's ratio is 0.35. The compressive yield strength is 100 MPa. The tensile specimens failed prior to yield at shrinkage induced surface flaws. As discussed in Adams and Wake (1984), the yield behavior of epoxy depends on both hydrostatic and deviatoric stress components. As a consequence, tensile yield strength is typically less than compressive yield strength. It seems reasonable to assume that the tensile yield strength of the epoxy used in this study is 80 MPa.

The butt tensile joint specimens were assembled using fixtures specially designed to ensure alignment of the two adherends and precise control of the adhesive bond thickness. Major components of the fixturing include a V-block, an aluminum holder, a digital micrometer, and a silicone boot. The V-block ensures axial alignment of the adherends; the aluminum holder holds the V-block secure and provides a mounting surface for the micrometer used to set bond thickness; and the silicone boot provides a seal to contain the epoxy adhesive in the bond gap during cure. The ends of the adherends were sandblasted and then cleaned and passivated in a sodium dichromate/nitric acid solution at room temperature (no electrical current was used) prior to bonding. The epoxy adhesive was mixed and vacuum evacuated to remove entrapped air prior to filling the bondline gap. A joint specimen's actual bond thickness was determined by calculating the difference between the final specimen length and the initial total length of the two adherends.

The butt tensile specimens were tested in a conventional, screw-driven load frame. The load train utilizes a chain linkage attached to the specimen via a pinned clevis. All specimens were loaded at a crosshead displacement rate of 0.5 in min^{-1}. Time to failure ranged between 10 and 20 s, depending on joint strength. The measured ratio of failure stress to time to fail varied from 1.9 to 2.6 MPa s^{-1}. Test results for all the butt tensile joints tested are listed in Tables 1–4. For each target bond thickness, joint strength displays moderate variability (standard deviation/average ratio ~10–15%). This variability is not large

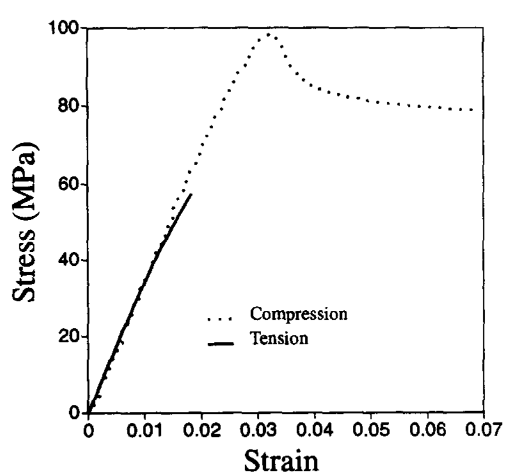

Fig. 2. Measured Epon 828/T-403 epoxy adhesive stress–strain relations.

Table 1. Target bond thickness = 0.25 mm

ID	$2h$ (mm)	σ^{ult} (MPa)	K_{fc} (MPa mm$^{0.32}$)
40	0.292	47.2	13.0
34	0.254	53.8	14.2
39	0.254	46.2	12.2
38	0.241	41.4	10.7
33	0.241	53.1	13.8
32	0.254	52.4	13.8
31	0.241	52.2	13.5
36	0.241	42.8	11.1
35	0.267	48.6	13.0
avg	0.254	48.6	12.8
s dev	0.017	4.6	1.2

Table 2. Target bond thickness = 0.50 mm

ID	$2h$ (mm)	σ^{ult} (MPa)	K_{fc} (MPa mm$^{0.32}$)
26	0.533	45.5	15.2
28	0.508	29.8	9.8
23	0.483	39.9	12.9
29	0.533	30.0	10.0
27	0.495	40.7	13.3
24	0.521	37.9	12.6
25	0.559	46.4	15.7
21	0.508	44.1	14.5
22	0.495	36.8	12.0
avg	0.515	39.0	12.9
s dev	0.024	6.1	2.1

Table 3. Target bond thickness = 1.0 mm

ID	$2h$ (mm)	σ^{ult} (MPa)	K_{fc} (MPa mm$^{0.32}$)
20	1.016	28.3	11.6
15	1.207	24.7	10.7
13	1.092	33.9	14.3
18	1.092	30.5	12.8
17	1.016	31.7	13.0
11	1.105	30.2	12.8
12	1.016	29.7	12.2
14	1.003	35.9	14.7
16	1.143	35.9	15.3
19	1.156	29.7	12.7
avg	1.085	31.0	13.0
s dev	0.070	3.5	1.4

Table 4. Target bond thickness = 2.0 mm

ID	$2h$ (mm)	σ^{ult} (MPa)	K_{fc} (MPa mm$^{0.32}$)
1	1.956	23.4	11.9
7	2.019	26.9	13.8
5	2.057	25.2	13.0
4	1.981	24.5	12.5
2	2.096	24.0	12.4
9	1.969	22.8	11.6
3	1.981	25.9	13.2
6	2.108	19.3	10.0
8	2.007	17.2	8.8
avg	2.019	23.2	11.9
s dev	0.056	3.1	1.6

enough, however, to obscure the pronounced dependence of measured joint strength on bond thickness.

A fractography examination of the failed joints indicates that failure always initiates adhesively (on the interface) along a small segment of the specimen periphery. This region of adhesive failure reaches no more than 5 mm into the specimen interior. Beyond this region, the failure is completely cohesive. These observations are consistent with a failure initiated by high interface corner stresses.

DISCUSSION

If butt tensile failure occurs when the interface corner stress intensity factor equals the interface corner fracture toughness K_{fc}, then using eqn (4)

$$\sigma_t^{ult} = \frac{(1-v)}{v}\frac{K_{fc}h^{\lambda-1}}{A_p(v)} \tag{8}$$

or

$$\log(\sigma_t^{ult}) = \log\left(\frac{(1-v)}{v}\frac{K_{fc}}{A_p(v)}\right) + (\lambda-1)\log(h). \tag{9}$$

Equation (9) indicates that the interface corner fracture toughness criteria predicts a linear relation between $\log(\sigma_t^{ult})$ and $\log(h)$ with slope $(\lambda-1)$. Recall that for the epoxy adhesive tested, $v = 0.35$ and $\lambda-1 = -0.32$. A log–log plot of measured butt tensile strength data

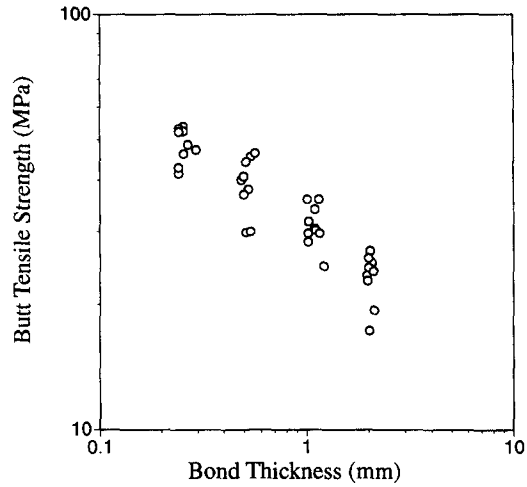

Fig. 3. Log–log plot of measured butt tensile strength as a function of bond thickness.

versus bond thickness (Fig. 3) also suggests a linear relation, and a least square fit of the measured data yields a slope of -0.35. The interface corner fracture toughness criterion accurately predicts the change in butt tensile strength with bond thickness.

Equation (4) can be used to calculate the interface corner fracture toughness associated with each joint test. Those values are listed in Tables 1–4, and plotted in Fig. 4 as a function of bond thickness. The values of K_{fc} do not vary with bond thickness in any systematic way. This suggests that K_{fc} is a material property and does not depend on bond thickness. The average value of the measured K_{fc} is 12.7 MPa mm$^{0.32}$. This value of K_{fc} is used in conjunction with eqn (8) to yield the butt tensile strength versus bond thickness prediction shown in Fig. 5 (the solid line). Prediction and measured data are in excellent agreement. Clearly, the test results reported here indicate that the interface corner fracture toughness criterion is valid, and can provide quite accurate predictions.

One interesting consequence of the interface corner fracture toughness criterion is the implied strength relationship between butt tensile joints with differing bond thicknesses [see eqn (4)]. Specifically

$$\sigma_{t1}^{ult}(h_1)^{1-\lambda} = \sigma_{t2}^{ult}(h_2)^{1-\lambda}, \tag{10}$$

where $2h_i$ is bond thickness, σ_{ti}^{ult} is the nominal butt tensile strength, and subscript $i = 1, 2$

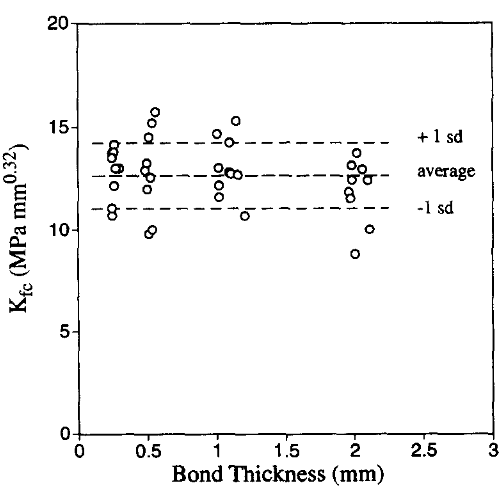

Fig. 4. Interface corner fracture toughness for all butt tensile joints tested (sd denotes standard deviation).

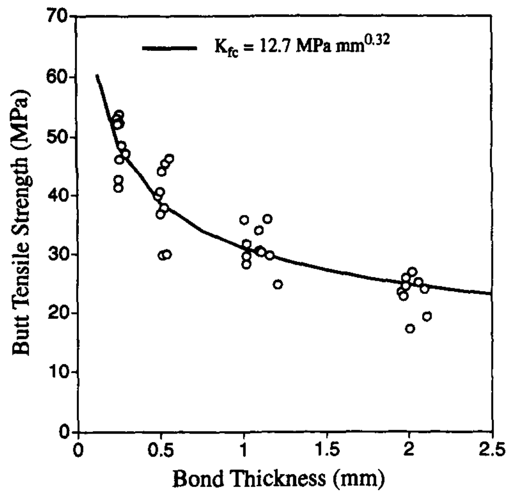

Fig. 5. Comparison of measured butt tensile strength vs bond thickness relation and interface corner fracture toughness-based estimate.

identifies the two joints with differing bond thickness. Since the value of $1-\lambda$ for Poisson's ratio values of 0.3–0.4 (values typical for most adhesives) ranges from 0.29 to 0.35, $1-\lambda$ can be reasonably approximated by the value of 1/3. Consequently, the strength of an adhesively bonded butt tensile joint of one thickness can be estimated from strength data for a joint with a different bond thickness by the simple relation

$$\sigma_{t2}^{ult} = \sigma_{t1}^{ult}(h_1/h_2)^{1/3}. \tag{11}$$

Finally, note that a necessary condition for the valid application of the interface corner fracture toughness criterion is the existence of small scale yielding conditions. Equation (11) applies only if bond thickness is much greater than the length of the interface corner yield zone. Equation (6) indicates that for the joints tested in this study $r_p = 0.005$ mm when the joint fails (for $K_{fc} = 12.7$ MPa mm$^{0.32}$, $\sigma_y = 80$ MPa). The ratio r_p/h for the thinnest bond tested has a value of less than 0.04.

CONCLUSIONS

The effect of bond thickness on butt tensile strength has been determined for a joint that bonds two stainless steel rods together with an unfilled epoxy adhesive (EPON 828/T-403). Joint strength doubles when bond thickness is reduced from 2.0 to 0.25 mm. A fracture analysis, based upon a critical interface corner stress intensity factor (i.e. on an interface corner fracture toughness), accurately predicts the observed bond thickness effect. This fracture criterion suggests that the strength of an adhesively bonded butt tensile joint of one bond thickness can be estimated from strength data for a joint with a different bond thickness by the simple relation

$$\sigma_{t2}^{ult} = \sigma_{t1}^{ult}(h_1/h_2)^{1/3},$$

where $2h_i$ is bond thickness, σ_{ti}^{ult} is the nominal butt tensile strength, and subscript $i = 1, 2$ identifies the two joints with differing bond thickness. This relation applies to thin bonds when the adhesive's Poisson's ratio is between 0.3 and 0.4, the adherends are relatively stiff compared to the adhesive, and small scale yielding conditions hold at the interface corner. The broader applicability of the interface corner fracture toughness criteria can only be determined by additional testing. The effect of adhesive fillers, residual stresses generated during cool down from an elevated temperature cure, and alternate joint geometries all merit consideration.

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